Math 284
Cuyamaca College

Name:________________________
Instructor: Dan Curtis

Practice Exam 2

1) True or False. Justify your answers. (Any matrix listed is **not** assumed to be square or invertible unless stated.)

a) If $AB = AC \text{ and } A \neq 0,$ then $B = C.$

b) If D is $n \times n$ and the equation $D\mathbf{x} = \mathbf{b}$ has no solution for some $b \in \mathbb{R}^n$, then D is not invertible.

c) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation such that $T(\mathbf{e}_1) = (3, 4)$ and $T(\mathbf{e}_2) = (-2, 7)$, then T is both one-to-one and onto.

d) Let A be a 3×5 matrix. Then the columns of A could be linearly independent, but they can't span $\mathbb{R}^3$.

2) a) Find the standard matrix, *A*, for the linear transformation given by:

$$T:\mathbb{R}^2 \to \mathbb{R}^3, T(\mathbf{x}) = \begin{bmatrix} 2x_1 - 3x_2 \\ x_2 \\ x_1 + 3x_2 \end{bmatrix}$$

b) Determine whether *T* is one-to-one, onto or both. Justify your answer.

3) Determine whether the given transformation is linear. Justify your conclusion.

$$T:\mathbb{R}^2 \to \mathbb{R}^2,\ T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 5x_1 + 2x_2 \\ 3x_2 \end{bmatrix}$$

4) For problem 4, let $A=\begin{bmatrix}1 & 2\\ -2 & 3\end{bmatrix}$, $B=\begin{bmatrix}2 & 3 & -3\\ 2 & -2 & 1\end{bmatrix}$, and $C=\begin{bmatrix}1 & 3 & 5\\ 2 & 4 & 6\\ 1 & 7 & 8\end{bmatrix}$. If the indicated calculation is not possible, indicate why.

a) Find *AB*.

b) Find *BA*

c) Find C^{-1}.

d) Find $\det(C)$

e) Find B^{T}

5) Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that reflects a vector across the x_1 axis and then rotates it counterclockwise by an angle of π. Find the standard matrix *A* for the transformation.

6) Suppose *A* and *B* are row equivalent $n \times n$ matrices, and the following series of row operations transforms *A* into *B*.

$R_1 \leftrightarrow R_3$

$-2R_1 + R_2$

$3R_1 + R_3$

$\frac{1}{4}R_2$

$2R_2 + R_3$

$6R_3$

If $B = \begin{bmatrix} -1 & 3 & 5 \\ 0 & 4 & 6 \\ 0 & 0 & 8 \end{bmatrix}$ find $\det(A)$.

7) Suppose D, E and F are invertible *n*x*n* matrices and *I* is the *n*x*n* identity matrix. Solve for E.

$$D^{-1}ED^{-1} + F = I$$

8) Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be the transformation $T(\mathbf{x}) = A\mathbf{x},$ where

$$A = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}.$$

Show on the graph the result of applying the transformation to the image below.